

As a result of the comments from both reviewers, we have revisited several parts of the paper. We found an easier to handle grammar for the full tie knots language, and as a result found an error in our proof of Theorem 2. In fact the deeper tucked language is context-free, not context-sensitive, and we are able to prove this with an explicit grammar and using the pumping lemma for regular languages.

As an effect of this new grammar, we were also able to use generating functions in all parts of the enumeration. We were unable (computationally) to extract a rational generating function for the full language, but could compute an initial series that allowed us to concretely enumerate all tie knots.

This additional work also unearthed an error in the previous numbers – 177 147 is a(n almost) correct count for singly tucked tie knots only if you allow tucks on the backside of the tie knot; the correct count is significantly lower. However, the full knot language is larger: we count 266 682 tie knots using the corresponding generating function.

Reviewer 1

Issue 1

The paper includes (long) tables of knots satisfying different constraints and a printout of the code used to produce these tables. This code has been put on Figshare, to ensure complete transparency. I do not know if PeerJ has a policy on the topic, but it would seem to me desirable to keep both code and paper tied for archival purposes.

Response:

We are happy to attach code and tables as supplementary materials to the paper on PeerJ to ensure everything is archived in a single place.

We are including files ties.py (the code) and tiestable.txt (the tables of knots) as supplemental files.

Issue 2

This paper looks like a worthwhile opportunity to include a closeup picture of a real knot. This would dramatically increase its accessibility.

Response:

We have taken closeup photos of three named and three randomly drawn tie knots. While the TeX-file (and PDF) currently use a single picture file with all six photos, we would prefer for production to use the single photos as included in the submission file set, and ordered in the commented TeX-code.

We have done it this way because of bugs in Overleaf.

Issue 3

p. 1 l. 18: The story presented here comes with no context. Who are those people, where did this take place? Knorr tells the story on his blog, but it is not clear where the original conversation took place.

Response:

Story drastically reduced – references to Knorr’s discussion instead, with a sentence-long summary.

Issue 4

p. 3, l. 95-96 “No region/direction shall repeat”: unclear phrasing

Response:

Clarifications added.

Issue 5

p.3 l. 96: It would be helpful to have a reminder somewhere, maybe here, of which symbol is inwards, which is outwards.

Response:

Symbol clarification added.

Issue 6

p.4 l. 123: As far as I can tell, theorem 1 does not exclude tucks that would be simultaneously k -fold or k' -fold (from just reading the sequence of windings). Is this correct? If so, does your enumeration distinguish this? Please clarify.

Response:

A tuck site can be simultaneously valid for a k -fold and a k' -fold tuck. We go into this in more detail in Section 4.2, after giving the recursive grammar. We have added a sentence to the text discussing Theorem 1 to clarify this.

Issue 7

In the cases where there are multiple tucks, you should probably say why it is not a concern that a tuck makes later tucks spanning more windings impossible (if I have two overlapping bows, and tuck under the top one, it makes it impossible later to tuck under the bottom one).

Response:

There are examples of knots that tuck deeper later after an earlier tuck. One example is the Allwin knot: <http://fairyfrog.deviantart.com/art/The-Allwin-tie-knot-Tutorial-433909390> where the deeper bow is accessible for tucks by spreading the tie winding.

Issue 8

p. 5 The rules after line 156 and line 157 are contradictory (in presentation only)

Response:

Fixed.

Issue 9

p. 6 l. 171: I think this automaton is wrong. It allows for the following sequence “TTTUTTTUTTTU”, which should not be allowed according to 4.1

Response:

Replaced with a new and less wrong automaton.

Issue 10

p. 7 line 206: finish the sentence with “... $k-2$ W and T symbols with proper counts”.

Response:

Fixed.

Reviewer 2**Issue 1**

As a matter of form, I would add that the first page is dominated by a verbatim history of the problem. It is unusual to me to see such a large amount of “fact” text copied from another source. Not all of this information is necessary, and I recommend the authors edit this section considerably.

Response:

Section considerably edited.

Issue 2

The proofs of theorems 3 and 4 are adhoc combinatorial arguments, but could be harmonized if more sophisticated enumeration tools were used. There are several formalisms which translate grammars into enumerative generating functions, and it seems like these should be a good candidate for such a process. Hence, from an enumeration point of view, it is lacking as the state of the art is more sophisticated. At the very least, they should offer an asymptotic comparison.

Response:

We would like to thank this reviewer for this suggestion – enumerative generating functions is a powerful tool for these questions. We were unaware of the connection to grammars, though it seems natural in hindsight. We have rewritten most of the enumeration, using generating functions in lieu of the previous ad hoc methods.

In this revision, Ingemar Markström was of great help, and performed a significant part of the generating functions calculations and proofs. We therefore have added him as a coauthor.

Issue 3

From a language theoretic point of view it is interesting that it is a context-sensitive language. Is there a similar argument for the Fink and Mao case?

Response:

Added text to Section 4.3 that specifically discusses the complexity class of Fink and Mao's language.

Issue 4

If the intended goal is to make new, usable patterns for ties, then the authors should add some focus on this– can they highlight some new knots that one could actually wear, (for which they provided have criteria).

Response:

Since building a theory of aesthetics for the facades is a far more difficult problem than the balancing features suggested by Fink and Mao, we have decided not to study these criteria beyond the summary of Fink and Mao's measurements.

Issue 5

The full tables at the back seem unnecessary given that they describe how to generate them.

Response:

The full tables are now extracted to a supplemental file.